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Savings Behavior of Return Migrants

A Life-Cycle Analysis* **

By Christian Dustmann

1. Introduction

Individuals' savings behavior has found considerable attention in the economic literature. Determinants of and motives for the accumulation of savings are typically analyzed in intertemporal models where agents maximize lifetime utility. The basic intertemporal model has been extended in various directions, for instance by the introduction of bequest motives (Yaari, 1964) or different tax regimes and credit constraints (Atkinson, 1971; Kahn, 1988). Research of this type concentrates on individuals who spend their entire life in one location. The savings behavior of individuals who change locations over their life cycle (usually referred to as migrants) has, with few exceptions, so far been neglected in the economic literature. This is surprising since migrants account for a considerable part of the population in many industrialized countries.[1] The savings of migrant populations are therefore of significant importance for both the countries of emigration and the countries of immigration.[2]

It has long been realized among social scientists that migrants often exhibit another savings behavior than natives do.[3] The savings behavior

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1 For instance, in 1990 workers of foreign nationality constituted 8.3% and 6.8% of the total workforce in Germany and France (see Velling and Woydt, 1993.)

2 Based on a representative sample from 1972, the *Bundesanstalt für Arbeit* reports that migrants to West Germany transferred between 30% and 45% of their disposable annual income to their home countries. Furthermore, a part of migrant households accumulated a considerable stock of savings in Germany (see *Monatsberichte der Deutschen Bundesbank*, April 1974).

3 For instance, Granier and Marciano (1975) find for France that the average local saving for foreign workers in 1970 was 50% higher than those of French workers with the same income. Kumcu (1989), using a survey conducted by the *Central Bank of the Republic of Turkey*, reports that for Turkish households in West-Germany the marginal propensity to save ranks between 0.21 and 0.48.

of migrants seems to be strongly related to the type of migration considered, and those migrants who only stay temporarily in the host country are often observed to have different savings pattern than native workers.[4] This temporary migration, or *return* migration, is in fact a major form of migration today. It can be observed in Europe and between European and non-European countries, in the United States and in Asia as well as between Asian countries and countries of the Middle East.

The importance of a future return of migrant workers to their home country on migrants' savings behavior has been emphasized by Djajic (1989) and Galor and Stark (1990), who show that migrants save more in the host country if the price level is lower or if wages are higher than in the home country. Galor and Stark (1991) and Dustmann (1994-b) show that lower wages in the home country affect also migrants' work effort in the host country. Karayalcin (1994) analyzes temporary migrations with immobile capital and shows that return migrants save more since they face a higher rate of interest. These contributions, however, assume the return point of the migrant as exogenous. Yet, if savings behavior of migrant workers is inherently related to their return plans, a theoretical analysis should endogenize return intentions.

This paper provides a theoretical analysis of the savings behavior of migrant workers where the simultaneity of savings- and return plans is taken explicitly into account. Djajic and Milbourne (1989) endogenize the return point by assuming that the marginal utility of consumption is higher in the host- than in the home region. A similar argument has been put forward by Hill (1981). Dustmann (1995) analyzes precautionary savings of migrants in such a framework where future income is uncertain. In this paper, two further reasons for an interior return point are examined which are likely to be relevant in real migration situations: different price levels, and human capital acquired abroad which enhances migrant's earnings position in the home country. An analysis of savings paths if a return is caused by any of these motives produces some interesting and unexpected results. Section 2 of the paper presents the basic model. In section 3, saving paths are analyzed in some detail. It is shown

Macmillan (1982) reports similar numbers concerning the saving behavior for migrants in other European countries. In an excellent and comprehensive survey on migration of Thai workers to countries of the Middle East Pitayanon (1986) reports that remittances of migrant workers are considerable and to a large proportion invested into savings.

4 Piore (1979, p. 54) emphasizes the accumulation of savings as a special feature of temporary migration. Paine (1974, p. 101) considers the saving of some target amount as the chief purpose of return migrants. Glytsos (1988) characterizes these migrants as *...staying relatively short periods of time in the receiving country, accumulating considerable amounts of money, remitting part of it during their stay abroad and returning home with the rest.*

that savings do not necessarily peak at the return point, as often suspected. Furthermore, if human capital considerations are responsible for a return, then savings profiles may peak even twice over the migrant's life cycle. Some simulations illustrate under what conditions which savings paths are likely to occur. The comparative statics of the model are then derived in section 4. In the case of endogenous determined return points, the paths of savings may be quite differently affected by changes in model parameters than with exogenous return decisions. The analysis has therefore various implications for empirical research.

2. The model

Consider a migrant worker at the beginning of his migration history in the host country. Denote his active lifetime, or remaining time in the labour force, with T. The migrant lives in a world of perfect foresight and with no uncertainty. At the point of immigration ($t = 0$), he decides about his future stream of consumption and determines how long to stay in the host country. Earnings per unit of time in the host country are exogenous. Earnings per unit of time in the home country after return, however, may depend on the time the migrant stayed abroad: the time spent in the host country may have enhanced the migrant's value for his home country economy. This is a simple formulation of human capital accumulation. It reflects that migrants from countries which are in the process of industrialization may acquire skills in an industrialized economy, like knowledge about working pattern and incentive structures, institutional features, language etc., which are important for the economy of the emigration country.[5]

Denote migrant's utility functional at any t as $u(c(t), \xi^i)$, $i = I, E$, where $c(t) \geq 0$ is a (time variant) flow of consumption and ξ is a location variable. The indices I and E denote immigration and emigration country respectively. The location variable reflects the migrant's *subjective perception* of characteristics of the environment, like climate, social relations etc. Indifference between locations should be reflected by location variables of equal size for both countries, while a preference for the home country is expressed by a higher location index for that region, relative to the host country. The location variables are assumed to be

[5] During the so-called guest-worker migration in Europe during the 50's, 60's and 70's, which drew approximately 10 Million workers from the periphery to the core countries, the accumulation of human capital was regarded as an important benefit by the countries of origin. Mehrländer (1980, p. 88) reports for guest-worker migration to Germany that the countries of origin expected out-migration *to improve the training of the workers concerned, ultimately creating a larger reservoir of skilled labour in the countries of origin.*

constant while residing in the respective region. The function u exhibits $u_c > 0$ and $u_{\xi^i} > 0$ and is strictly concave with respect to both arguments c and ξ^i, $i = I, E$. Furthermore, the cross derivatives $u_{c\xi^i}$ are strictly positive: the marginal utility of consumption is increasing in the location variables. This formulation follows Hu (1978) and Djajic and Milbourne (1988), who generate optimal stopping points by complementary variables in the utility function which take different values in different stages over the life cycle. For simplification, the following notation will be used: $u(k, \xi^E) = v^E(k)$, $u(k, \xi^I) = v^I(k)$. Let the utility functions have the properties that $\lim_{k \to \infty} v^{i\prime} = 0$ and $\lim_{k \to 0} v^{i\prime} = \infty$, where the superscripts$'$ denotes first derivatives. Throughout the analysis, it will be assumed that the home country location is evaluated by the migrant at least as high as the host country location ($\xi^I \leq \xi^E$). It then follows straightforwardly that utility from a constant flow of consumption k is at least as high in the home- than in the host country both in marginal and in absolute terms: $v^E(k) \geq v^I(k)$); $v^{E\prime}(k) \geq v^{I\prime}(k)$. The migrant's objective function over the horizon T is thus given by:

$$(1) \qquad J = \int_0^{\hat{t}} v^I(c(\tau))\, e^{-\rho\tau}\, d\tau + \int_{\hat{t}}^{T} v^E(c(\tau))\, e^{-\rho\tau}\, d\tau,$$

where $\hat{t}$ is the point of return and ρ the rate of time preference. The migrant maximizes the functional (1) with respect to consumption c and the return point $\hat{t}$, subject to the following budget constraint:

$$(2) \qquad \begin{aligned} &\int_0^{\hat{t}} c^I(\tau)\, e^{-r\tau}\, d\tau + \int_{\hat{t}}^{T} p c^E(\tau)\, e^{-r\tau}\, d\tau - \int_0^{\hat{t}} y^I e^{-r\tau}\, d\tau \\ &- \int_{\hat{t}}^{T} y^E(\hat{t})\, e^{-r\tau}\, d\tau - K_0 + \bar{K} e^{-rT} = 0, \end{aligned}$$

where c^I and c^E are the flows of consumption abroad and at home, respectively. Further, r is the (time-constant) interest rate and p is the relative price level between emigration- and immigration country. The stock of savings at $t = 0$ is denoted by K_0 and the stock of capital at the end of the planing horizon by $\bar{K}$. Earnings per unit of time in the immigration- and the emigration country are given by y^I and y^E. It will be assumed throughout the analysis that the initial wage differential is positive: $y^I > y^E$ for $t = 0$. As mentioned above, y^E may positively depend on the time abroad. In this case, it is convenient and natural to assume that y^E is a concave function in $\hat{t}$: $y^E_{\hat{t}} > 0$, $y^E_{\hat{t}\hat{t}} \leq 0$.

The above optimization problems is solved in two stages. In the first stage, optimal consumption- and savings plans will be chosen for any $\hat{t}$. Denote the marginal utility of wealth at t as $\pi(t)$, with $\pi(t) = \pi^0 e^{(\rho - r)t}$, where π^0 is the marginal utility of wealth at $t = 0$. Furthermore, denote savings out of wage income in the immigration- and emigration country by $S^I(t)$ and $S^E(t)$, respectively, with:

$$(3) \qquad S^I(t) = [y^I - c^I(t)]; \; S^E(t) = [y^E(\hat{t}) - pc^E(t)].$$

Optimality of the path of consumption requires that:

$$\pi(t) = \begin{cases} v^{I\prime}(c(t)) & : 0 \le t < \hat{t} \\ \frac{1}{p} v^{E\prime}(c(t)) & : \hat{t} \le t \le T. \end{cases}$$

Inversion of (4) and using (3) yields:

$$(5) \qquad S(t) = \begin{cases} S^I(t) = y^I - v^{I\prime -1}(\pi(t)) & : 0 \le t < \hat{t} \\ S^E(t) = y^E - v^{E\prime -1}(\pi(t)p)p & : \hat{t} \le t \le T, \end{cases}$$

where the superscript $^{-1}$ denotes inverse functions. Savings at each point in time t are a function of $\pi(t)$. Over the life cycle, the path of savings develops as follows:

$$(6) \qquad \dot{S}(t) = \begin{cases} \dot{S}^I(t) = \frac{1}{-v^{I\prime\prime}} \dot{\pi} & : 0 \le t < \hat{t} \\ \dot{S}^E(t) = \frac{1}{-v^{E\prime\prime}} \dot{\pi} p & : \hat{t} \le t \le T. \end{cases}$$

Obviously, because $\dot{\pi} = (\rho - r)\pi^0 e^{(\rho - r)t}$ and the utility functions are strictly concave, savings increase over time if $\rho > r$, and decrease if $\rho < r$. Throughout the analysis, only that case will be considered where the rate of time preference is smaller than the interest rate: $\rho < r$.[6] Remember that the location indices ξ may differ between home- and host country, reflecting different preferences for the respective environment. Since the marginal utility of consumption is increasing in the location variable, a shift of consumption at the point of return may possibly take place: the consumption profile shifts upwards at $t = \hat{t}$ if $\xi^I < \xi^E$, for $p = 1$. Even if the migrant is indifferent between host- and home coun-

[6] To assume that individuals are highly impatient ($\rho > r$) implies some complications of the analysis. Further restrictions are necessary to ensure a unique optimal solution. In the literature, this case is usually excluded, mainly because it does not seem to correspond to observed consumption patterns. For an exposition of the complications arising in a life cycle model if $\rho < r$, see Blinder and Weiss (1976).

try environment, the path of consumption will exhibit a discontinuity at the point of return if the price level differs between countries: $p \neq 1$. For a higher price level in the host country, this will likewise result in an upward shift of the consumption profile at the return point. Formally, this may be expressed as:

$$\lim_{t \to \hat{t}^-} (c^I(t)) \leq \lim_{t \to \hat{t}^+} (c^E(t)) \text{ as } v^{I\prime}(k)\, p \leq v^{E\prime}(k). \tag{7}$$

The path of the stock of savings at any t is given by the following expression:

$$K(t) = \begin{cases} K_0\, e^{rt} + \int_0^t e^{r(t-\tau)} S^I(\pi(\tau))\, d\tau : 0 \leq t < \hat{t} \\ K(\hat{t})\, e^{rt} + \int_{\hat{t}}^t e^{r(t-\tau)} S^E(\pi(\tau))\, d\tau : \hat{t} \leq t \leq T. \end{cases} \tag{8}$$

Expression (8) fully characterizes the migrant's consumption plan over the life cycle, for a given $\hat{t}$ and the corresponding π^0.

In the second stage of the optimization problem, that consumption plan as a function of the optimal point of return, $\hat{t}$, will be chosen which maximizes (1) under the constraint (2). Using (5), the budget constraint (2) implicitly determines π^0 as a function of $\hat{t}$:

$$\int_0^{\hat{t}} S^I(\tau)\, e^{-r\tau}\, d\tau - \int_{\hat{t}}^{T} S^E(\tau)\, e^{-r\tau}\, d\tau + K = \Gamma(\hat{t}, \pi^0), \tag{9}$$

with $K = K_0 - \bar{K} e^{-rT}$. Differentiating the utility function and the budget constraint with respect to the return point $\hat{t}$ and combining terms using (3) and (4) yields:

$$[v^I - v^E] + \pi^0\, e^{(\rho - r)\hat{t}} \left[[S^I - S^E] + \frac{1}{r} y_{\hat{t}}^E [1 - e^{r(\hat{t}-T)}] \right] = \Delta(\hat{t}, \pi^0), \tag{10}$$

where all expressions are evaluated at $t = \hat{t}$.[7]

The first term in (10) is the marginal cost of staying one unit of time longer in the host country, in terms of forgone utility: staying longer abroad deprives the migrant of the possibility to consume during that unit of time in the home country. The second term is the marginal benefit of lengthening the time abroad. Note that costs and benefits are both measured in units of utility. The marginal benefit of staying longer abroad has two components. It allows the migrant to accumulate more

[7] Moreover, note from (4) and (5) that for expressions carrying superscript I this evaluation must be obtained by taking the limes $\lim_{t \to \hat{t}}$. For notational convenience this is suppressed here and in the following.

resources $[S^I - S^E]$ for lifetime consumption, and it increases his potential earnings back home, given $y_{\hat{t}}^E > 0$. The optimal point of return to the home country is then characterized as equating the marginal costs and benefits discussed above – hence, $\Delta = 0$.

Relations (9) and (10) are two equations in two unknowns, π^0 and $\hat{t}$. There are three scenarios for which an interior return point exists:

(a) $\xi^E > \xi^I, y_{\hat{t}}^E = 0, p = 1$;

(b) $\xi^E = \xi^I, y_{\hat{t}}^E = 0, p < 1$;

(c) $\xi^E = \xi^I, y_{\hat{t}}^E > 0, p = 1$.

In scenario (a), the relative price level is equal in both countries, and the time in the host country does not enhance the migrant's earnings potential back home. However, the migrant appreciates his home country location more than the host country location. This preference may induce him to return before retirement age. This is essentially what triggers a return in Djajic and Milbourne (1988). In scenario (b), the migrant is indifferent between locations, but prices abroad are higher than at home. A temporary migration may occur because it is advantageous for the migrant to exploit high wages abroad and low prices at home. In scenario (c), price levels are again equal in both countries and the migrant is indifferent between locations, but he enhances his earnings potential in the home country by the mere fact of staying longer abroad.[8] Necessary and sufficient conditions for an interior solution in these three cases are given in the Appendix. Observed situations of return migration are likely to be generated as combinations of the above three scenarios.

The analysis below distinguishes between return situations which are generated by a combination of scenario (a) and scenario (b) only, and situations generated by all three scenarios. Technically, both (a) and (b) ensure that the marginal benefit of staying abroad increases to a lower extent than the marginal cost, thus inducing the term Δ to decrease. Migration occurs if $\Delta_{\hat{t} \to 0} > 0$. Return occurs when marginal benefit equals marginal cost, or $\Delta = 0$. With only (c), the marginal cost of staying abroad longer is constant and equal to zero, since the migrant is indifferent between consuming at home and abroad. However, the marginal benefit is decreasing over time, and again an interior solution occurs when $\Delta = 0$.[9]

[8] For an interior return point to occur in scenario (c) it is necessary that potential earnings in the home country eventually overcome actual earnings in the host country for some t, $t \in (0, T)$. See the appendix for a characterization of this solution.

[9] For a more detailed analysis of areturn under these three scenarios, see Dustmann (1994-a).

3. Savings profiles of temporary migrants

Return migrants are often observed to accumulate savings while being abroad and to spend after return. This savings pattern is commonly understood as characterizing return migration (see, for instance, Paine (1974), Piore (1979) and Glytsos (1988)). However, a peak in the savings profile at return is only one possible savings pattern consistent with utility maximization. Migrants' savings profiles may well peak before or after the return point. Moreover, the analysis points out that a peak in savings stocks at the return point is not a peak in the sense of the classical life cycle model. Furthermore, under certain circumstances, the savings profile of the migrant may be double peaked. This occurs if working experience abroad enhances the migrant's earnings position in his home country. All these results are derived below and illustrated by simulations.[10]

Recall equation (8) which characterizes the migrant's stock of savings path. It follows that savings around the point of return are given by:

$$\text{(11-a)} \qquad \lim_{t \to \hat{t}} \dot{K}(t) = rK(\hat{t}) + S^I(\hat{t}),$$

$$\text{(11-b)} \qquad \lim_{t \to \hat{t}^+} \dot{K}(t) = rK(\hat{t}) + S^E(\hat{t})$$

Assume for the moment that $y_{\hat{t}}^E = 0$. In other words, a return is induced by a combination of scenarios (a) and (b). It follows from (7) and (10) that at the optimal return point $t = \hat{t}$, $[v^E - v^I] > 0$ and $\Delta(\hat{t}, \pi^0) = 0$. Therefore, it must be that at this point $S^E(t) = [y^E - c^E(t)p] < S^I(t) = [y^I - c^I(t)]$. This condition is compatible with three saving schemes:

$$\text{(I):} \quad S^E(\hat{t}) = [y^E - c^E(\hat{t})p] < 0, S^I(\hat{t}) = [y^I - c^I(\hat{t})] < 0; \qquad S^E(\hat{t}) < S^I(\hat{t}).$$

$$\text{(II):} \quad S^E(\hat{t}) = [y^E - c^E(\hat{t})p] < 0, S^I(\hat{t}) = [y^I - c^I(\hat{t})] > 0; \qquad S^E(\hat{t}) < S^I(\hat{t}).$$

$$\text{(III):} \quad S^E(\hat{t}) = [y^E - c^E(\hat{t})p] > 0, S^I(\hat{t}) = [y^I - c^I(\hat{t})] > 0; \qquad S^E(\hat{t}) < S^I(\hat{t}).$$

Under scheme (I), the migrant's stock of savings peaks while he is in the host country. Towards the end of his migration history, his flow of consumption is higher than his flow of income. After return, the migrant continues to reduce his stock of savings, but at a higher rate. Figure (1)

[10] For simulation purposes, the utility functions are specified as $v^i(c(t)) = \frac{1}{1-\beta}\xi^i c(t)^{1-\beta}, i = I, E$. The accumulation function for earnings abroad is specified as $y^E(\hat{t}) = \bar{y}^E + \gamma \ln(1 + \hat{t}); \gamma > 0$.

shows simulated consumption-earnings profiles for this case and figure (2) depicts the path of the stock of savings.[11]

Under scheme (III), the migrant continues to accumulate savings even after returning to his home country. Savings peak after the return point, but the rate of accumulation of savings decreases after the return point. This case is illustrated in figure (3).

The classical savings profile of return migrants is generated by scheme (II). The migrant saves while he is abroad. Savings peak at the point of return, and the migrant reduces his stock of savings over his remaining cycle. Figure (4) illustrates this case. Notice that a peak in savings corresponding to the peak in conventional life cycle models occurs only in situations described by (I) and (III): with scheme (I) $S^E(t) = 0$ for $t < \hat{t}$ and with scheme (III) $S^I(t) = 0$ for $t > \hat{t}$. It follows from (9) that both situations are indeed characterized by a unique peak before or after remigration. Under scheme (II), however, a flow of consumption which is equal to the flow of income is never realized. This is a direct consequence of the discontinuity of the consumption flow at the return point. A peak in savings profiles of return migrants upon return is therefore not a peak in the sense of the classical life cycle model.

Consumption – Earnings Profiles

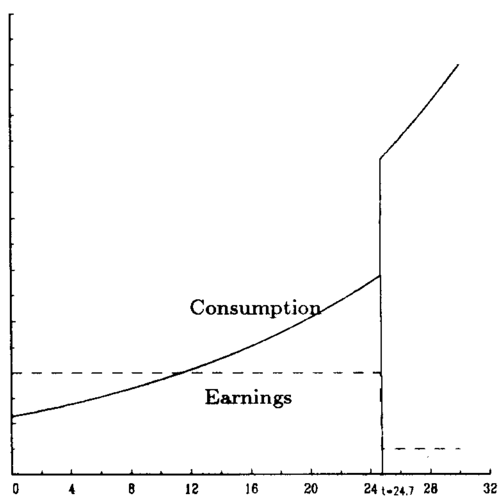

Figure 1: Consumption – Earnings Profiles.
$[y^E = \xi^I = 1; y^I = 4; \xi^E = 1.2; \hat{t} = 24.7; \pi^0 = 0.72.]$

[11] In all the following simulations, ρ, r, β, T and p are set to 0.08, 0.1, 0.4, 30 and 1 respectively.

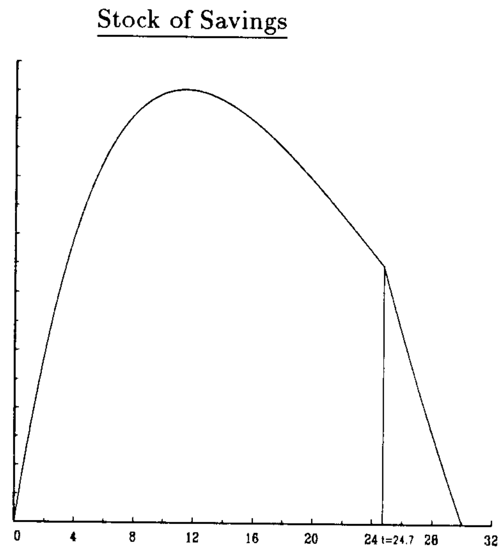

Figure 2: Peak of Savings Stock before Return.

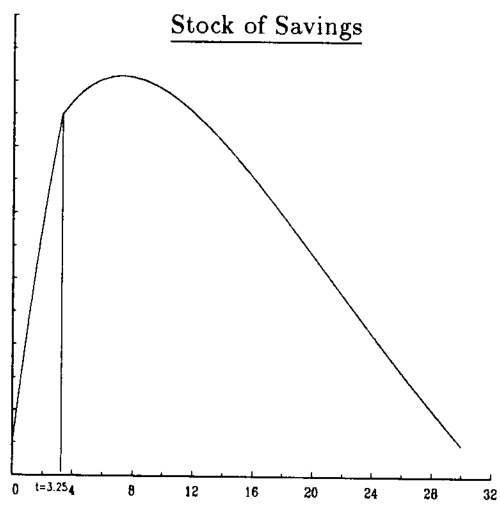

Figure 3: Peak of Savings Stock after Return.
[$y^E = \xi^I = 1$; $y^I = \xi^E = 1.2$; $\hat{t} = 3.25$; $\pi^0 = 1.38$.]

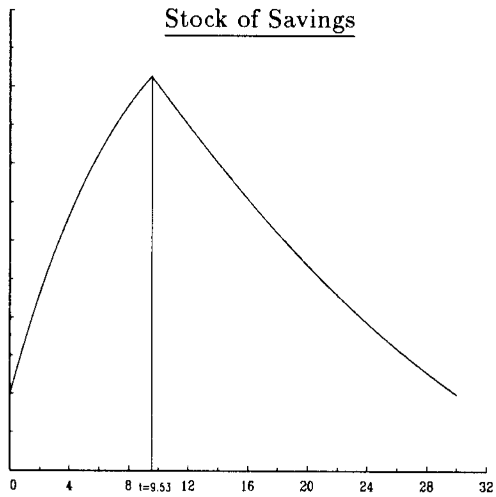

Figure 4: Peak of Savings Stock at Return.
[$y^E = \xi^I = 1$; $y^I = 2$; $\xi^E = 1.5$; $\hat{t} = 9.53$; $\pi^0 = 1.28$.]

An important question is how likely these schemes are to occur in real migration situations. A situation like that under scheme (II) seems most compatible with empirical evidence. To gain further insight into which parameter constellation generates which type of scheme, consider some simulations. Firstly, let a return be induced solely by scenario (a). Assume that w^E, ξ^I and p are equal to one. Let the index ξ^E vary over the range $[1, 2]$, where 1 corresponds to indifference between locations. At the same time, let the wage rate abroad, w^I, vary between 1 and 10. A value of 10 would correspond to a wage in the host country which is by factor 10 higher than the wage in the home country. Simulations are now performed by calculating for each combination of ξ^E and w^I the savings scheme which will occur. Results are illustrated by figures (5) and (6).

In figure (5), the vertical axis carries the environment index ξ^E and the horizontal axis the wage rate in the host country w^I. The lines partition the two-dimensional plane in three areas, where the numbers (I), (II) and (III) refer to the different schemes which are obtained at the respective combinations of ξ^E and w^I. It is obvious that scheme (I) occurs in situations which are characterized by very small wage differentials, and scheme (III) in situations which are characterized by small differences in the environment index. Accordingly, migrations compatible with scheme (I) should be of short duration, while migrations compatible with scheme (III) should be of long duration. Intermediate cases are compatible with

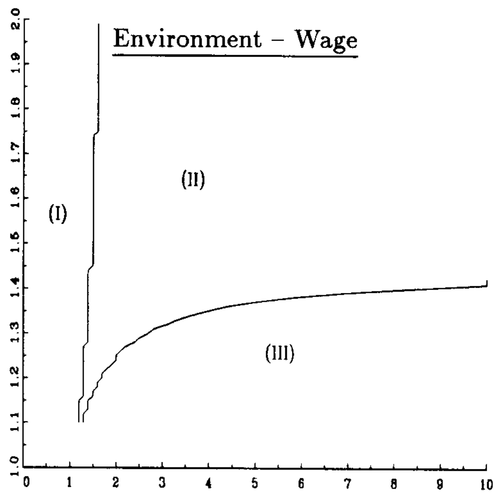

Figure 5: Horizontal Axis: w^I; Vertical Axis: ξ^E.

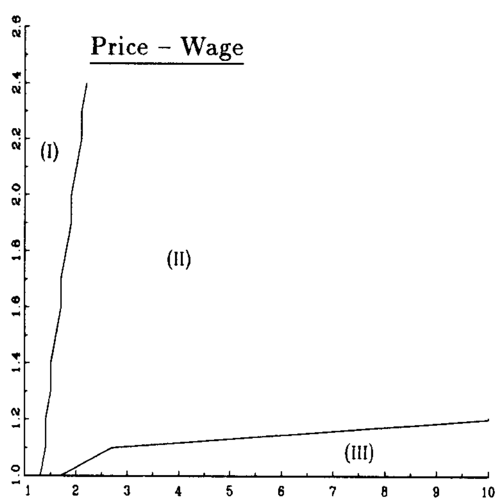

Figure 6: Horizontal Axis: w^I; Vertical Axis: p.

the typically observed savings scheme (II): migrants accumulate savings while being abroad, and reduce savings after return.

A similar picture emerges if return is triggered by price levels only. In figure 6, the vertical axis carries the relative price level p, where the price level in the home country is set to one and the price level abroad is varied between 1 and 2.5. The horizontal axis carries the wage abroad again, which takes values between 1 and 10. Furthermore, $w^E = \xi^E = 1$, and ξ^I is set to 1.2. Again, schemes (I) and (III) occur only for very small wage differentials, or very small price differentials, respectively. Most migration situations are likely to be characterized by intermediary wage-environment or wage-price combinations, which are compatible with scheme (II). Nevertheless, savings behavior as represented by schemes (I) and (III) is perfectly consistent with rational behavior.

Savings and Human Capital

If a return is induced by a combination of (a), (b) and (c), then the savings profile of the migrant may well peak twice over the life cycle, a first time in the host country, and a second time in the home country. At the return point, this scheme is characterized as follows:

$$\text{(IV)} \quad S^E(\hat{t}) = [y^E(\hat{t}) - pc^E(\hat{t})] > 0, \; S^I(\hat{t}) = [y^I - c^I(\hat{t})] < 0; \; S^E(\hat{t}) > S^I(\hat{t}).$$

It is easy to see that this case violates the necessary condition for an optimum if a return were induced by scenarios (a) and (b) only. Since $S^E(\hat{t}) > S^I(\hat{t})$, $\Delta(\hat{t}, \pi^0) = 0$ is only fulfilled for $[v^I - v^E] > 0$, which is in contradiction with the assumptions in (a) and (b) (see (10)). Nevertheless, it is possible that this situation occurs if scenario (c) is allowed for. To see this, recall that with (c), $y^E_t > 0, y^E_{tt} \leq 0$. It follows then from (10) that at $t = \hat{t}$ situation (IV) may occur, as long as the following holds:

$$(12) \qquad [S^E - S^I]\pi + [v^E - v^I] = \frac{1}{r} y^E_t (1 - e^{r(\hat{t} - T)})\pi.$$

Relation (12) says that the migrant will return to his home country when the increase in his wage potential at home by spending a further unit of time abroad (RHS of (12)) is equal to the sum of the net increase in the stock of savings and the forgone utility of not being able to consume during this unit of time at home. Since the RHS of (12) is positive, a situation may occur where $S^E > S^I$ and $v^E \geq v^I$ at $t = \hat{t}$.

Under scheme (IV), the migrant's desired consumption supersedes his income towards the end of his migration history, and he starts to reduce his stock of savings ($S^I(t) = 0$ for $t < \hat{t}$). Upon return, the migrant's earning situation improves (this follows directly from (12) and the strict concavity of the utility function). If desired consumption rises to a lower extent, he again begins to accumulate savings until consumption supersedes earnings in the home country also. In figure (7), the respective

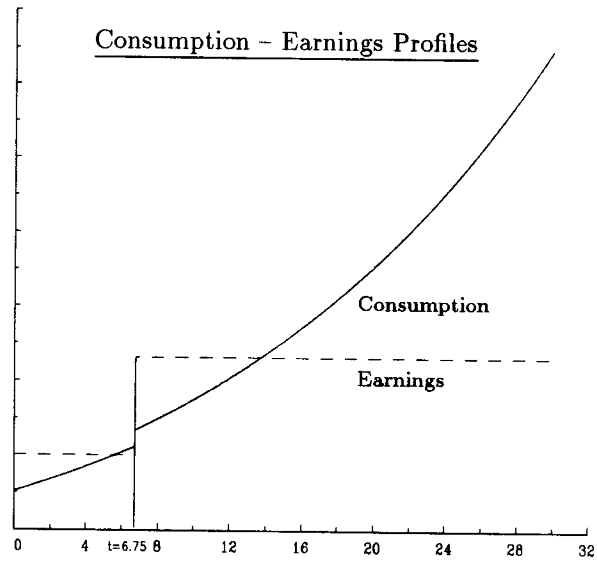

Figure 7: Consumption – Earnings Profiles, Double – Peaked Savings Stock.

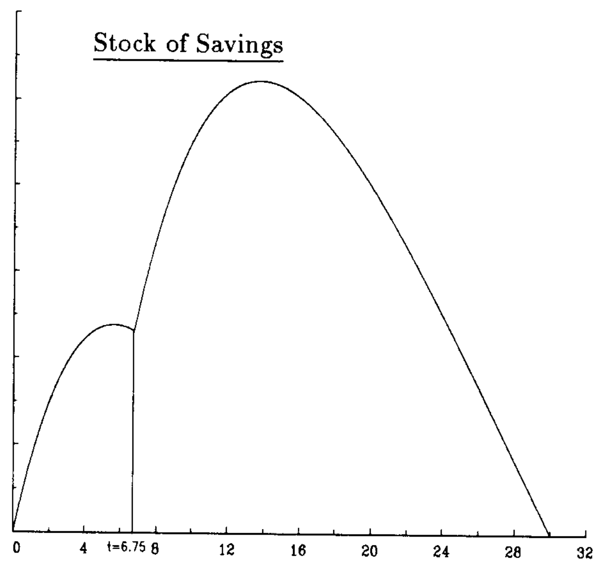

Figure 8: Double – Peaked Savings Stock.

consumption- and earnings profiles are illustrated. The discontinuity in income at the return point induces a second peak in savings over the life cycle, which is demonstrated in figure (8).

4. Comparative statics

In a life-cycle model with an exogenous return point, comparative statics are relatively straightforward. Changes in parameters affect savings at any time t by changing the marginal utility of wealth, π^0 (see (5)). For a given $\hat{t}$, π^0 is determined by equation (9). If the return point is endogenous, however, the analysis becomes slightly more complicated: a change in parameters does not only affect the marginal utility of wealth π^0 *directly* (via (9)), but also *indirectly* by changing the return point (via (10)). As shown below, this additional effect may change qualitative results. Therefore, the endogenization of return intentions is particularly important if results should provide a guidance for empirical work, where data are generated by individuals who meet both consumption- and return decisions simultaneously.

It is helpful to consider the way in which any change in parameter x affects savings in country i, $i = E, I$:

$$\text{(13)} \qquad \frac{dS^i}{dx} = \left[\frac{dy^i}{dx} - \frac{dc^i}{dx}p - \frac{dc^i}{d\pi^0}\frac{d\pi^0}{dx}p\right] - \left[\frac{dc^i}{d\pi^0}\frac{d\pi^0}{d\hat{t}}\frac{d\hat{t}}{dx}\right]p,$$

where $p = 1$ for $i = I$. The first term in brackets contains the usual effects of parameter changes on savings: it is the sum of effects on earnings per unit of time and the direct and indirect effects on the flow of consumption. If the return point is endogenous, however, a second indirect effect occurs which is due to changes in the flow of consumption, induced by changes of the optimal return point (second term in brackets). It is this second effect which causes comparative static results to be different for migration situations with endogenous and exogenous return points.

To perform comparative statics requires analyzing the equations (9) and (10) simultaneously at the point where $\Gamma(\hat{t}, \pi^0) = \Delta(\hat{t}, \pi^0) = 0$. In the appendix, it is shown that around this point a unique local differentiable solution exists, so that the implicit function theorem is applicable. Comparative statics on π^0 and $\hat{t}$ are likewise derived in the appendix. Since the utility function is strictly concave, consumption is a strictly decreasing function in the marginal utility of wealth, π^0: $(dc^i/d\pi^0) < 0$.

Again, consider first the case where a return is induced only by scenarios (a) and (b) and combinations thereof. Table 1 shows some qualitative effects of changes in respective variables on savings at any point in time in the host country (column (I)) and in the home country (column (II)). The last column reports the change in the marginal utility of wealth as a reaction on changes in respective parameters. Notice that $\pi^0_{\hat{t}}$ is equal to zero if return points are exogenous, so that in this case the second term in the numerator diminishes. Notice further that $D > 0$.

Table 1

Comparative Statics, Endogenous Return Point

x	$\frac{dS^I}{dx}$	Sign	$\frac{dS^E}{dx}$	Sign	$\frac{d\pi^0}{dx} + \frac{d\pi^0}{d\hat{t}}\frac{d\hat{t}}{dx}$	Sign
	I		II		III	
y^I	$1 - \frac{\partial c^I}{\partial \pi^0}\frac{\partial \pi^0}{\partial y^I}$	$\gtreqless 0$	$-\frac{\partial c^E}{\partial \pi^0}\frac{\partial \pi^0}{\partial y^I}p$	< 0	$\frac{\partial \pi^0}{\partial y^I} = \frac{{\pi^0}_{y^I} + {\pi^0}_{\hat{t}}\,\hat{t}_{y^I}}{D}$	< 0
y^E	$-\frac{\partial c^I}{\partial \pi^0}\frac{\partial \pi^0}{\partial y^E}$	$\gtreqless 0$	$1 - \frac{\partial c^E}{\partial \pi^0}\frac{\partial \pi^0}{\partial y^E}p$	$\gtreqless 0$	$\frac{\partial \pi^0}{\partial y^E} = \frac{{\pi^0}_{y^E} + {\pi^0}_{\hat{t}}\,\hat{t}_{y^E}}{D}$	$\gtreqless 0$
p	$-\frac{\partial c^I}{\partial \pi^0}\frac{\partial \pi^0}{\partial p}$	$\gtreqless 0$	$-p\frac{\partial c^E}{\partial \pi^0}\frac{\partial \pi^0}{\partial p} - c^E$	$\gtreqless 0$	$\frac{\partial \pi^0}{\partial p} = \frac{\pi^0_p + {\pi^0}_{\hat{t}}\,\hat{t}_p}{D}$	$\gtreqless 0$
K^0	$-\frac{\partial c^I}{\partial \pi^0}\frac{\partial \pi^0}{\partial K^0}$	< 0	$-\frac{\partial c^E}{\partial \pi^0}\frac{\partial \pi^0}{\partial K^0}p$	< 0	$\frac{\partial \pi^0}{\partial K^0} = \frac{\pi^0_K \frac{\partial K}{\partial K^0}}{D}$	< 0
$\bar{K}$	$-\frac{\partial c^I}{\partial \pi^0}\frac{\partial \pi^0}{\partial \bar{K}}$	> 0	$-\frac{\partial c^E}{\partial \pi^0}\frac{\partial \pi^0}{\partial \bar{K}}p$	> 0	$\frac{\partial \pi^0}{\partial \bar{K}} = \frac{\pi^0_K \frac{\partial K}{\partial \bar{K}}}{D}$	> 0

The effects of the initial stock of savings, K^0, and the desired stock of savings out of labour income at the end of the planning period, $\bar{K}$, on savings are unambiguous: the higher K^0, the lower are savings in home- and host country; the higher $\bar{K}$, the higher are savings in both countries. Changes in $\bar{K}$ and K^0 effect only directly the scarcity of wealth over the migrant's life cycle, and they therefore have a unique effect on π^0. These results hold for endogenous and exogenous return points.

The more interesting cases are those which occur as a result of changes of wages abroad or at home, and in changes of the relative price level.[12] Consider first an increase of wages abroad, y^I. This raises consumption in both countries and decreases unambiguously savings in the home country. The effect on savings in the host country is unclear, and it depends on the specific assumptions made about the utility function. This effect may well change sign over the migration cycle. For exogenous return points, one obtains the same qualitative effects.

[12] Comparative statics with respect to y^E refer to changes in the base wage the migrant receives without possible gains from human capital acquired abroad. With scenario (c), one may think of $y^E(\hat{t})$ as $y^E(\hat{t}) = y^E + f(\hat{t})$.

One should expect that an increase in wages in the home country, y^E, has likewise a positive effect on consumption in both countries. However, this is only the case if the return point is exogenous. Then an increase in the wage rate decreases the marginal utility of wealth, which, in turn, raises consumption flows at home and abroad. If the return point is endogenous, an additional effect occurs: the optimal duration abroad decreases ($\pi_{\hat{t}}^0 \hat{t}_{y^E} > 0$), and this *increases* the marginal utility of wealth (see column III, table 1). As a result, the total effect of an increase in home country wages on consumption is ambiguous.[13] Of course, effects on savings are indeterminate as well.

This is an unexpected and important result. A direct consequence for empirical research is that, when estimating consumption functions of migrants, the analyst should have different expectations for parameter estimates, depending on whether return decisions are taken inside or outside the model. In particular, if estimating consumption functions of return migrants in the host country, a *negative* coefficient on a variable which represents wages in the home country should be entirely consistent with rational behavior.

Now consider changes in the relative price level p. As in conventional life cycle models, an increase in the relative price level generally exhibits no clear-cut effect on the consumption- and savings behavior at home and abroad. The reason for this is that changes in p induce counteracting income- and substitution effects. The same ambiguity applies to exogenous returns. Only in the case where $\pi_p^0 < 0$ (which requires $c^E < \pi p \frac{\partial c^E}{\partial p}$, see appendix), an increase in the relative price level decreases savings at home and abroad unambiguously.

Allowing for Human Capital Accumulation

In the preceding analysis, a return is induced by (a) and/or (b) only. Now (c) can also induce the migrant's return. In this case, the results of the comparative statics change only for the home country. In particular, it introduces an additional term in (13): $(dy^E/d\hat{t})\,(d\hat{t}/dx)$ is the effect of a change in the desired duration abroad, induced by changes of any parameter x, on the earnings position in the home country. The effect of changes in respective parameters on consumption or savings follows straightforwardly from the appendix. Obviously, the results for consumption will not change, but all the effects on savings will in this case be ambiguous.

[13] Notice that both effects occur also for changes in y^I, but they point in the same direction.

Consider now the case where a return is induced *only* by scenario (c). The individual is indifferent between locations ($\xi^E = \xi^I$) and the price level is the same in both countries ($p = 1$). Migration is temporary only because the time abroad enhances the migrant's potential earnings in his home country, so that he will return when $[y^E - y^I] - \frac{1}{r} y_{\hat{t}}^E (1 - e^{r(\hat{t} - T)})] = 0$ (see equation (9)). In this case, $\pi_{\hat{t}}^0 = 0$ (see appendix): changes in the optimal time abroad do not affect the marginal utility of income. This reflects the migrant's complete indifference between locations. In this situation, qualitative effects of changes in parameters on consumption and savings are identical to those obtained for exogenous return points.

5. Summary and Conclusions

This paper analyzes the savings behavior of migrant workers in a life cycle model where the return to the home country is an endogenous choice variable. Migrants return home because they have a preference for their home country location, the relative price level is higher abroad, and/or the time the migrant spends in the host country enhances his earnings position at home.

Although it is widely believed that the savings stock of a typical return migrant peaks at the point of return to his home country, the analysis points out that this need not always be the case. Saving stocks of return migrants which peak before or after the return point are entirely consistent with utility maximizing behavior. Nevertheless, comparative statics based on simulations show that a parameter constellation which generates a peak at return is most likely to occur in real migration situations. Furthermore, if one motive to return is that human capital acquired abroad enhances migrant's earnings potential in his home country, then his saving profiles may even peak twice, one time in the host country, and a second time in the home country.

The endogeneity of the return decision changes the comparative statics of the model. Parameter changes have a second indirect effect on consumption. One immediate result is that an increase in migrant's wages at home will not increase consumption flows unambiguously, as a model with exogenous return points would predict. Thus, for empirical work on migrants' savings behavior the analysis suggests carefully examining under which institutional restrictions the data at hand were generated. In situations where migrants are free to decide about their return, the appropriate theoretical framework to be used as a guide for an empirical specification should model return- and savings behavior simultaneously.

Furthermore, the appropriate empirical model should explicitly account for the endogeneity of return decisions.

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Appendix

Necessary condition for $\hat{t}, \pi^0$ being an optimal solution to (9) and (10) is that $\Gamma(\pi^0, \hat{t}) = 0$ and $\Delta(\pi^0, \hat{t}) = 0$. Sufficient for a maximum is that Γ is increasing in π^0 for any admissible $\hat{t}$ and Δ is decreasing in $\hat{t}$ for any admissible π^0. In other words, the Jakobian of the system described by (9) and (10) has to fulfill the saddle point conditions, with $\partial\Gamma/\partial\pi^0 > 0$ and $\partial\Delta/\partial\hat{t} < 0$. Denote the Jakobian with H:

$$H = \begin{bmatrix} \frac{\partial \Gamma}{\partial \pi^0} & \frac{\partial \Gamma}{\partial \hat{t}} \\ \frac{\partial \Delta}{\partial \pi^0} & \frac{\partial \Delta}{\partial \hat{t}} \end{bmatrix} \tag{14}$$

The respective elements are easily derived by differentiating the system (9) and (10):

$$\frac{\partial \Gamma}{\partial \pi^0} = -\left[\int_0^{\hat{t}} \frac{\partial c^E}{\partial \pi} e^{(\rho - 2r)\tau} dt + \int_{\hat{t}}^{T} p \frac{\partial c^I}{\partial \pi} e^{(\rho - 2r)\tau} dt\right] > 0 \tag{15}$$

$$\text{since } \frac{\partial c}{\partial \pi} = \frac{1}{v_{11}} < 0,$$

$$\frac{\partial \Gamma}{\partial \hat{t}} = \frac{\partial \Delta}{\partial \pi^0} = [(S^I - S^E) + \frac{1}{r} y_{\hat{t}}^E (1 - e^{r(\hat{t} - T)})] e^{-r\hat{t}} \geq 0, \tag{16}$$

$$\frac{\partial \Delta}{\partial \hat{t}} = \dot{\pi}[(S^I - S^E) + \frac{1}{r} y_{\hat{t}}^E (1 - e^{r(\hat{t}-T)})] + + \pi[y_{\hat{t}\hat{t}}^E (1 - e^{r(\hat{t}-T)}) - y_{\hat{t}}^E (1 + e^{r(\hat{t}-T)})] < 0. \tag{17}$$

The signs for (16) and (17) follow straightforwardly from the necessary condition $\Delta = 0$. Consider first situation (a): $\xi^E > \xi^I$, $p = 1$ and $y_{\hat{t}}^E = 0$. It follows from (7) that $[v^I - v^E] < 0$ and, therefore, from (10) that $[S^I - S^E] > 0$. The same line of argumentation holds for situation (b), which is described by $\xi^E = \xi^I$, $p < 1$ and $y_{\hat{t}}^E = 0$. In both situations, the expression in (15) is strictly positive. In situation (c), characterized by $\xi^E = \xi^I$, $p = 1$ and $y_{\hat{t}}^E > 0$, $[S^I - S^E] + \frac{1}{r} y_t^E (1 - e^{r(t-T)}) = 0$ for the optimal $\hat{t}$. Accordingly, the expression in (16) is equal to zero. Since $y_{\hat{t}}^E$ is concave in $\hat{t}$, the expression in (17) is always negative. Note that, for situation (c), $\xi^I = \xi^E$ and $p = 1$ and, therefore, $v^I - v^E = 0$ and $c^I = c^E p$ so that $y^I + \frac{1}{r} y_t^E [1 - e^{r(t-T)}] = y^E$. Consequently, it is necessary for an interior solution that there exists some $t \in (0, T)$ for which the wage differential is reversed: the profile of the potential wage in the home country crosses the (constant) wage profile in the host country.

Since $Det(H) \neq 0$, the implicit function theorem may be applied to derive comparative static results. For this purpose, first totally differentiate (9) with respect to $\hat{t}$, π^0, p, y^E, y^I, ξ^I, ξ^E, K and T. This results in the following expression:

$$\begin{aligned} d\hat{t} &= \frac{a_1}{a} d\pi^0 + \frac{a_2}{a} dy^E + \frac{a_3}{a} dy^I + \frac{a_4}{a} dp + \frac{a_5}{a} dK + \frac{a_6}{a} d\xi^I + \frac{a_7}{a} d\xi^E + \frac{a_8}{a} dT \\ &= \underset{(+)}{\hat{t}_{\pi^0}} d\pi^0 + \underset{(-)}{\hat{t}_{y^E}} dy^E + \underset{(+)}{\hat{t}_{y^I}} dy^I + \underset{(+)}{\hat{t}_p} dp + \underset{(0)}{\hat{t}_K} dK + \underset{(+)}{\hat{t}_{\xi^I}} d\xi^I + \underset{(-)}{\hat{t}_{\xi^E}} d\xi^E + \underset{(+)}{\hat{t}_T} dT, \end{aligned} \tag{18}$$

where

$$a = \dot{\pi}[S^I - S^E + \frac{1}{r} y_{\hat{t}}^E (1 - e^{r(\hat{t}-T)})] + \pi[y_{\hat{t}\hat{t}}^E (1 - e^{r(\hat{t}-T)}) - y_{\hat{t}}^E (1 + e^{r(\hat{t}-T)})] < 0,$$

$$a_1 = e^{(\rho - r)\hat{t}}[S^E - S^I - \frac{1}{r} y_{\hat{t}}^E (1 - e^{r(\hat{t}-T)})] \leq 0,$$

$$a_2 = \pi > 0,$$

$$a_3 = -\pi < 0,$$

$$a_4 = -\pi c^E < 0,$$

$$a_5 = 0,$$

$$a_6 = -\frac{\partial v^I}{\partial \xi^I} < 0,$$

$$a_7 = \frac{\partial v^E}{\partial \xi^E} > 0,$$

$$a_8 = -\pi[y_{\hat{t}}^E e^{r(\hat{t}-T)}] < 0.$$

Totally differentiating (10) and re-arranging terms yields:

$$d\pi^0 = \frac{b_1}{b}d\hat{t} + \frac{b_2}{b}dy^E + \frac{b_3}{b}dy^I + \frac{b_4}{b}dp + \frac{b_5}{b}dK + \frac{b_6}{b}d\xi^I + \frac{b_7}{b}d\xi^E + \frac{b_8}{b}dT$$

$$= \underset{(-)}{\pi^0_{\hat{t}}}\,d\hat{t} + \underset{(-)}{\pi^0_{y^E}}\,dy^E + \underset{(-)}{\pi^0_{y^I}}\,dy^I + \underset{(?)}{\pi^0_p}\,dp + \underset{(-)}{\pi^0_K}\,dK + \underset{(+)}{\pi^0_{\xi^I}}\,d\xi^I + \underset{(+)}{\pi^0_{\xi^E}}\,d\xi^E + \underset{(+)}{\pi^0_T}\,dT, \quad (19)$$

where

$$b = -\left[\int_0^{\hat{t}} \frac{\partial c^E}{\partial \pi} e^{(\rho-2r)\tau}\,dt + \int_{\hat{t}}^{T} p\frac{\partial c^I}{\partial \pi} e^{(\rho-2r)\tau}\,dt\right] > 0$$

$$\text{since } \frac{\partial c}{\partial \pi} = \frac{1}{v_{11}} < 0,$$

$$b_1 = [S^E - S^I] - \frac{1}{r}y^E_{\hat{t}}(1 - e^{r(\hat{t}-T)})]e^{-r\hat{t}} \leq 0$$

$$b_2 = -\left[1 - e^{-r(T-\hat{t})}\right]\frac{1}{r}e^{-r\hat{t}} < 0,$$

$$b_3 = -\left[1 - e^{-r\hat{t}}\right]\frac{1}{r} < 0,$$

$$b_4 = \int_{\hat{t}}^{T} [c^E + \pi p\frac{\partial c^E}{\partial p}]e^{-r\tau}\,d\tau \gtreqless 0 \text{ as } c^E \gtreqless \pi p\frac{\partial c^E}{\partial p},$$

$$b_5 = -1 < 0,$$

$$b_6 = \int_0^{\hat{t}} \frac{\partial c^I}{\partial \xi^I} e^{-r\tau}\,d\tau > 0 \text{ since } \frac{\partial c^I}{\partial \xi^I} = \frac{\frac{-\partial v^I_1}{\partial \xi^I}}{v^I_{11}} > 0,$$

$$b_7 = \int_{\hat{t}}^{T} p\frac{\partial c^E}{\partial \xi^E} e^{-r\tau}\,d\tau > 0 \text{ since } \frac{\partial c^E}{\partial \xi^E} = \frac{\frac{-\partial v^E_1}{\partial \xi^E}}{v^E_{11}} > 0,$$

$$b_8 = [pc^E - y^E]e^{-rT} > 0 \text{ for } pc^E > y^E \text{ at } t = T.$$

Notice that b_1 equals zero if a return is induced only by scenario (c). Therefore, in this case the term $\pi^0_{\hat{t}}$ vanishes.

Rewrite (18) and (19):

$$\begin{bmatrix} 1 & -\hat{t}_{\pi_0} \\ -\pi^0_{\hat{t}} & 1 \end{bmatrix}\begin{bmatrix} d\hat{t} \\ d\pi^0 \end{bmatrix} = \begin{bmatrix} \hat{t}_{y^E}dy^E & \hat{t}_{y^I}dy^I & \hat{t}_p dp & \hat{t}_K dK & \hat{t}_{\xi^I}d\xi^I & \hat{t}_{\xi^E}d\xi^E & \hat{t}_T dT \\ \pi^0_{y^E}dy^E & \pi^0_{y^I}dy^I & \pi^0_p dp & \pi^0_K dK & \pi^0_{\xi^I}d\xi^I & \pi^0_{\xi^E}d\xi^E & \pi^0_T dT \end{bmatrix}$$

Using Cramer's rule, the partial effects in table 1 follow straightforward, where $D = 1 - \hat{t}_{\pi^0}\pi^0_{\hat{t}} = 1 - \frac{a_1 b_1}{a b} > 0$ follows directly from (14).